Problem Set #1

(due 10/22/19)

1. Consider an economy in which relative producer prices are fixed and a representative household, with a unit endowment of labor, maximizes the following utility function:

$$U(c_1, c_2, l) = (c_1 - a_1)^{\beta_1}(c_2 - a_2)^{\beta_2} l^{1-\beta_1-\beta_2}$$

(where c_1 and c_2 are consumption goods and l is leisure), subject to the budget constraint:

$$p_1 c_1 + p_2 c_2 + wl = w$$

A. Derive an explicit solution (i.e., in terms of prices and preference terms a_i and β_i) for the excess burden of taxes on c_1, c_2, and l as a function of the original, undistorted prices of the three goods (p_1^0, p_2^0, and w^0), the distorted prices (p_1^1, p_2^1, and w^1), and a fixed utility level.

B. Show that excess burden equals zero if $p_i^1 = (1+\theta)p_i^0$, i = 1, 2, and $w^1 = (1+\theta)w^0$ for some constant θ.

C. Compare the values of excess burden based on utility levels achieved in the absence and in the presence of taxation, $V(p_1^0, p_2^0, w^0)$ and $V(p_1^1, p_2^1, w^1)$.

D. Using the measure derived in part A, show that the marginal excess burden for an increase in a tax *or* subsidy on good 2 is positive. (*Hint*: relate the change in excess burden to the sign of $(p_2^1 - p_2^0)$.)

2. Suppose that an economy has two goods, education and housing, and that every family has preferences over the two goods defined by the common utility function, $U(E, H) = E^{\alpha}H^{1-\alpha}$. Households differ only with respect to income level, with household i's exogenous income equal to y^i. Housing is produced subject to constant unit cost $p_H = 1$, and may be purchased in any quantity. Education may be produced using one of two technologies: by the private sector as a regular private good with unit cost $p_E = 1$ per family, and by the public sector as a *pure public good* with unit cost q per unit of the common level of public education. Publicly provided education is financed by a proportional tax at rate τ on income, and no individual household may use public and private education at the same time.

A. For fixed values of the tax rate, τ, and the level of public education, G, show that there exists a critical level of income, $\hat{y}$, above which households choose private school, and below which households choose public school. Show that $\hat{y}$ is increasing in G, given τ.

B. Start with your solution for $\hat{y}$ as a function of G and τ from part A. Letting Y equal aggregate income in the economy, substitute for τ using the government's budget constraint that relates τ to q, G, and Y. Calculate the full effect of G on $\hat{y}$, i.e., the effect taking into account the impact of G on τ. Show that this effect is larger than the partial effect you solved for in part A, and explain why.

C. Show that, among individuals who choose public education, there is a single level of public education, say G^*, that is most preferred by all, given preferences and the use of proportional taxation to pay for public education. If the existing level of public education is initially at G^*, under what condition would a majority of the overall population vote for a small decrease in spending on public education spending? (*Hint*: relate $\hat{y}$ at G^* to the income of the median voter.)

3. Consider an economy with two commodities, X and Y, each produced competitively using a single factor, labor, which is in fixed overall supply, $L = L_x + L_y$. Producers of X have the production function $X = (\alpha e^{-\beta Y})L_x$ and producers of Y have the production function $Y = \gamma L_y$. That is, each producer perceives constant returns to scale with respect to labor, but labor productivity in sector X faces a negative externality based on the aggregate production of Y.

   A. Derive the economy's production possibilities frontier as an expression for X in terms of Y, and show that the production set is not convex, i.e., that there are linear combinations of feasible production bundles that are not themselves feasible.

   B. Letting labor be numeraire, derive the cost functions for producers of each good, $c(X;Y)$ and $c(Y)$, assuming that producers of X take Y as given and that producers of Y ignore their impact on sector X. Solve for the competitive prices at given values of X and Y.

   C. Now, solve for the social cost function for X and Y, $c(X,Y)$, and the marginal social costs of X and Y at given production levels. Derive expressions for Pigouvian taxes on producers of X and Y that would cause competitive prices to equal marginal social costs.

   D. Suppose that consumers are identical, with preferences that satisfy the utility function $U(X,Y) = X+Y$. How much revenue does the Pigouvian tax raise at the social optimum?

4. In the Harberger two-sector model, labor bears 100% of an excise tax on sector-X output if the ratio of capital income to gross income (including the excise tax) is unchanged.

   A. For the same assumptions as in the standard Harberger model (e.g., fixed overall supplies of labor and capital, no initial distortions), show that this outcome requires that sector X be more labor intensive than sector Y.

   B. Using expressions from Lecture Note 6, derive a condition that depends only on factor shares (θ), factor allocations (λ) and elasticities of substitution (σ) for labor to bear at least 100% of an excise tax in sector X.

   C. Assume that sector X is more labor intensive than sector Y, so that (from the result in part A) it is possible for labor to bear 100% of an excise tax on sector X. Using the expression you derived in part B, show that, in the limit as goods X and Y become perfect substitutes in consumption (i.e., as $\sigma_D \rightarrow \infty$), labor must bear at least 100% of the tax.